Disclosure, Privacy Models, and Privacy Mechanisms

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V. Torra (2022) A guide to data privacy, Springer (Chapter 3)

Outline

- Privacy models
    - Definition
    - Summary
    - Privacy from re-identification
    - k-Anonymity
    - Differential privacy
    - Homomorphic encryption
    - Secure multiparty computation
    - Result privacy
    - Summary
    - An example: Integral privacy

Introduction

Definition

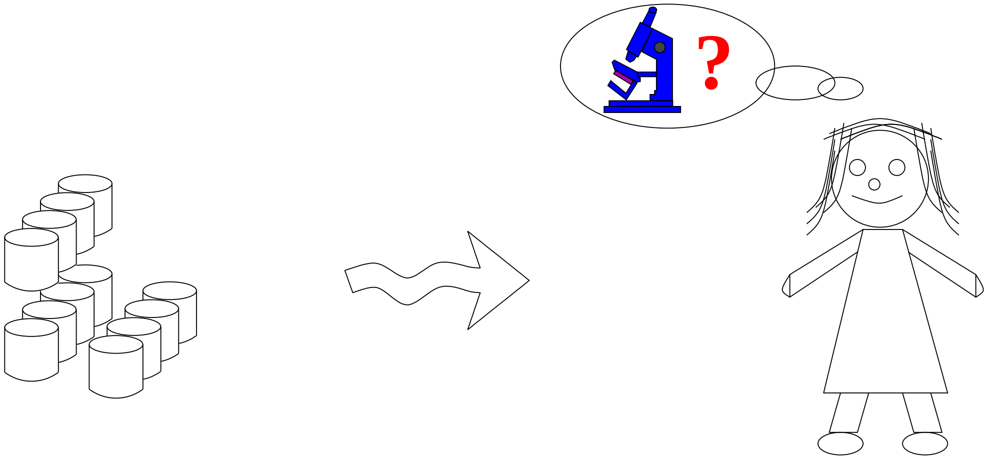
Disclosure

Definition

- A privacy model is a computational definition of privacy.

Summary of privacy models

Privacy models



Privacy models

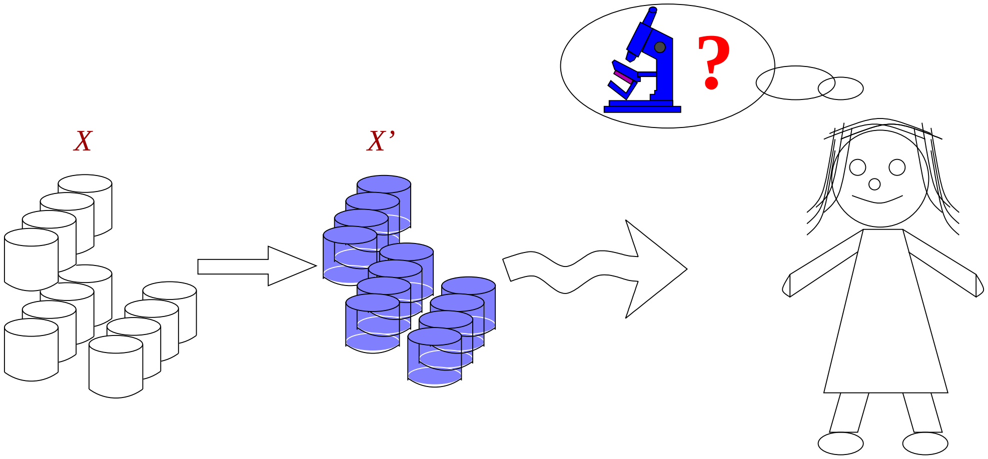
Privacy models. A computational definition for privacy. Examples.

- **Reidentification privacy.** Avoid finding a record in a database.
- **k-Anonymity.** A record indistinguishable with $k-1$ other records.
- **Secure multiparty computation.** Several parties want to compute a function of their databases, but only sharing the result.
- **Differential privacy.** The output of a query to a database should not depend (much) on whether a record is in the database or not.
- **Result privacy.** We want to avoid some results when an algorithm is applied to a database.
- **Integral privacy.** Inference on the databases. E.g., changes have been applied to a database.
- **Homomorphic encryption.** We want to avoid access to raw data and partial computations.

Privacy models

Privacy models. A computational definition for privacy. Publish a DB

- **Reidentification privacy.** Avoid finding a record in a database.
- **k-Anonymity.** A record indistinguishable with $k-1$ other records.
- **k-Anonymity, l-diversity.** l possible categories
- **Interval disclosure.** The value for an attribute is outside an interval computed from the protected value: values different enough.
- **Result privacy.** We want to avoid some results when an algorithm is applied to a database.



Privacy models

Privacy models. A computational definition for privacy. Publish a DB

- Modify DB X to obtain a DB X’ compliant with the privacy model.

Original DB X:

Respondent	City	Age	Illness
DRR	Barcelona	30	Heart attack
ABD	Barcelona	32	Cancer
COL	Barcelona	33	Cancer
GHE	Tarragona	62	AIDS
CIO	Tarragona	65	AIDS
HYU	Tarragona	60	Heart attack

Published DB X':

———	City	Age	Illness
—	Barcelona	30	Cancer
—	Barcelona	30	Cancer
—	Barcelona	30	Cancer
—	Tarragona	60	AIDS
—	Tarragona	60	AIDS
—	———	–	———

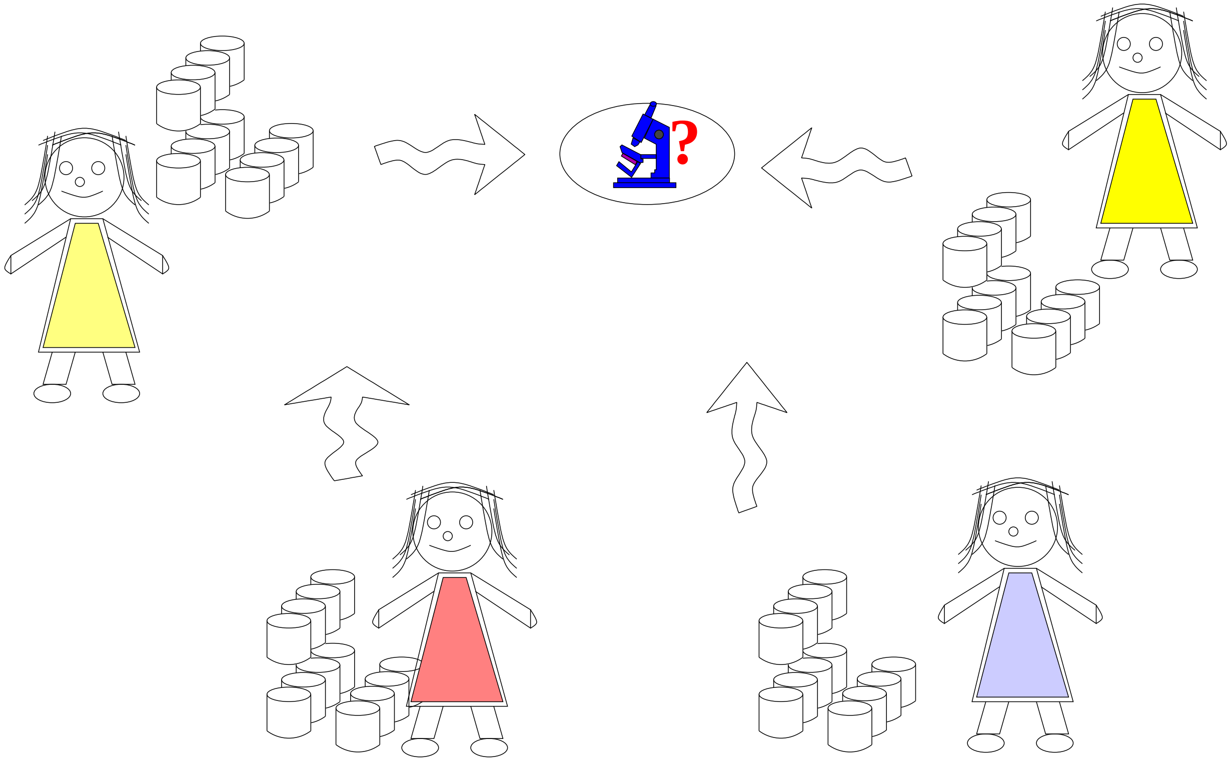
Privacy models

- Difficulties
  Naive anonymization does not work, highly identifiable data, high dimensional data
- Examples of successful reidentification attacks
  Sweeney analysis of USA population, data from mobile data, shopping cards, film ratings

Privacy models

Privacy models. A computational definition for privacy. Share a result

- **Secure multiparty computation.** Several parties want to compute a function of their databases, but only sharing the result.



Privacy models

Privacy models. A computational definition for privacy. Share a result

- Compute
$$f(DB_1, DB_2, DB_3, DB_4)$$
without sharing DB_1, DB_2, DB_3, DB_4
- Example: national age mean of hospital-acquired infection patients (hospitals do not want to share the age of their infected patients!)

Privacy models

- Difficulties

  Distributed approach (no trusted-third party) – computational cost of solutions

Privacy models

Privacy models. A computational definition for privacy. Compute result

- **Differential privacy.** The output of a query to a database should not depend (much) on whether a record is in the database or not.
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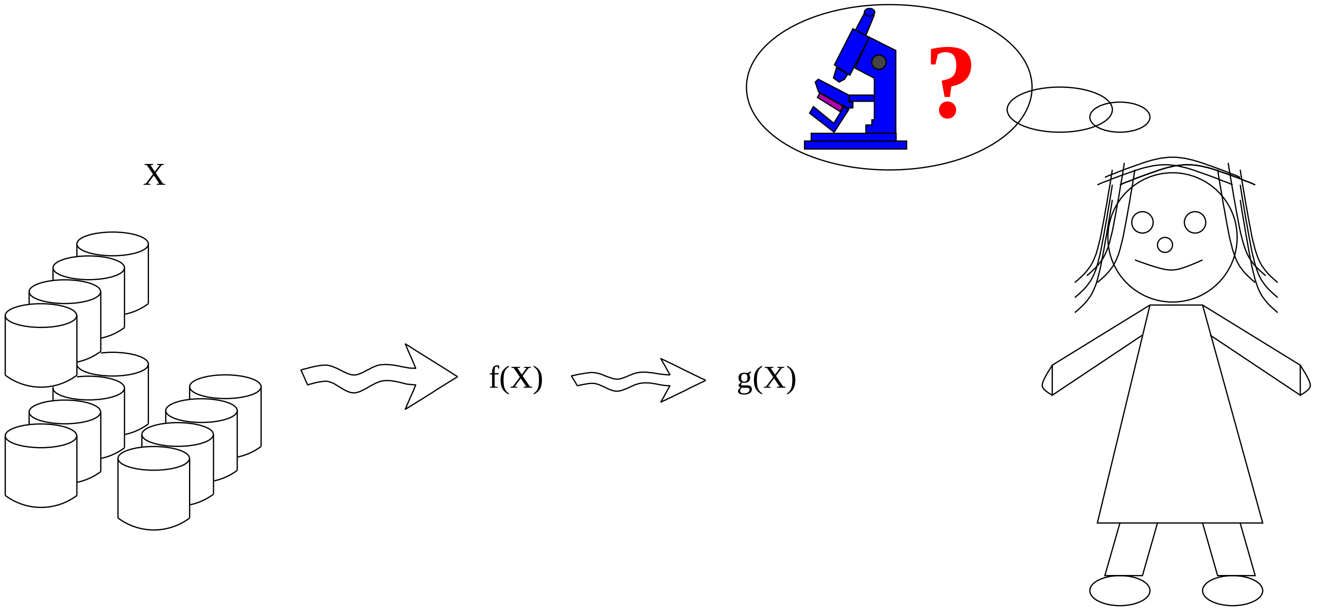

Privacy models

- Difficulties. A simple function can give information on who is in the database
  - E.g., mean salary

Privacy from re-identification

Privacy from re-identification

- A protected database A satisfies privacy from re-identification given intruder's knowledge B when

$$Reid(B, A) \leq r_{R1}$$

with a certain privacy level r_{R1} (e.g., $r_{R1} = 0.25$),

- or, alternatively

$$KR.Reid(B, A) \leq (r_K, r_{R1})$$

with certain privacy levels r_K and r_{R1} (e.g., $r_K = 0$ and $r_{R1} = 0.5$).

k-**Anonymity**

k-Anonymity

Definition 3.4

- A database A satisfies k-anonymity with respect to a set of quasi-identifiers QI when the projection of A in this set QI results into a partition of DB in sets of at least k indistinguishable records.

City	Age	Illness
Barcelona	30	Cancer
Barcelona	30	Cancer
Tarragona	60	AIDS
Tarragona	60	AIDS

k-Anonymity

- Indistinguishability w.r.t. quasi-identifiers
- k-Anonymity and re-identification

$$KR.Reid(B, A) \leq (0, 1/k).$$

- Plausible deniability

k-Anonymity

- Indistinguishability w.r.t. quasi-identifiers
- k-Anonymity and re-identification

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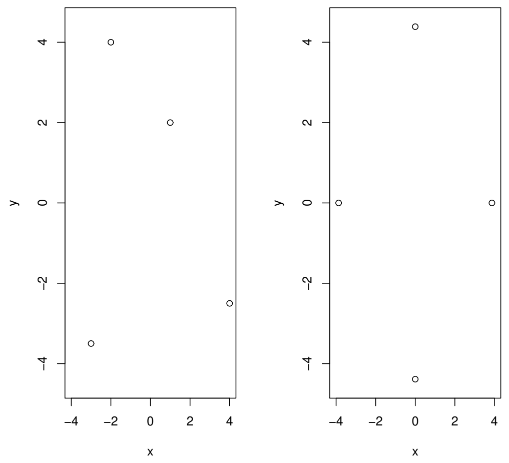
- Plausible deniability
  - ○ at record level
  - ○ but not at database level
- Records are independent

k-Anonymity

- k-confusion. Drop indistinguishability

- Example
  - Original data: $X = \{(1,2),(-2,4),(4,-2),(-3,-3)\}$.
  - k-Anonymity: $X' = \{(0,0),(0,0),(0,0),(0,0)\}$.
  - k-Confusion: using $X'' = \{(x,0),(-x,0),(0,y),(0,-y)\}$, with standard deviations in X'' equal to the ones in X
    - ▷ $x = \sqrt{10}/\sqrt{2/3} = 3.872983$, $y = \sqrt{12.8333}/\sqrt{2/3} = 4.387476$



- Discussion: k-confusion and re-identification

k-Anonymity

- Attacks
  - Homogeneity attack (external attack)
  - External knowledge attack (internal attack)
- These are attribute disclosure attacks
  - while k-anonymity is for identity disclosure
- Variations of k-anonymity to avoid attribute disclosure

k-Anonymity

- p-sensitive k-anonymity for $k > 1$ and $p \leq k$
  - if it satisfies k-anonymity and, for each group of records with the same combination of values for a set of quasi-identifiers, the number of distinct values for each confidential value is at least p (within the same group).
- l-diversity
  - forces l different categories in each set. However, in this case, categories should have to be <u>well-represented</u>. Different meanings have been given to what <u>well-represented</u> means.
- t-closeness.
  - The distribution of the attribute in any k-anonymous subset of the database is similar to the one of the full database. Similarity: distance

between the two distributions, distance below a given threshold t. The Earth Mover distance is used in the definition.

k-Anonymity

- k-anonymity and computational anonymity
  - Relaxation: not-all quasi-identifiers
    "We say that unconditional anonymity is theoretical anonymity. Computational anonymity is conditioned by the assumption that the adversary has some limitation. The limitations can be (...) restricted memory or knowledge." (Stokes (2012)).
  - A data set X satisfies (k,l)-anonymity if it is k-anonymous with respect to every subset of attributes of cardinality at most l.
    $\Rightarrow$ Intruder's knowledge limited to l attributes

- Example: (2,2)-anonymity

$$D = \{(a,b,e),(a,b,f),(c,d,e),(c,d,f),$$

$$(c,b,e),(c,b,f),(a,d,e),(a,d,f)\}.$$

Differential privacy

Differential privacy

- Computation-driven/single database
  - Privacy model: differential privacy[1]
  - We know the function/query to apply to the database: f

- Example:

  compute the mean of the attribute salary of the database for all those living in Town.

[1] There are other models as e.g. query auditing (determining if answering a query can lead to a privacy breach), and integral privacy

Differential privacy

- Differential privacy (Dwork, 2006).
  - Motivation:
    - ▷ the result of a query should not depend on the presence (or absence) of a particular individual
    - ▷ the impact of any individual in the output of the query is limited

  differential privacy ensures that the removal or addition of a single database item does not (substantially) affect the outcome of any analysis (Dwork, 2006)

Differential privacy

- Mathematical definition of differential privacy (in terms of a probability distribution on the range of the function/query)
  - A function K_q for a query q gives ϵ-differential privacy if for all data sets D_1 and D_2 differing in at most one element, and all $S \subseteq Range(K_q)$,

$$\frac{Pr[K_q(D_1) \in S]}{Pr[K_q(D_2) \in S]} \leq e^{\epsilon}.$$

(with 0/0=1) or, equivalently,

$$Pr[K_q(D_1) \in S] \leq e^{\epsilon} Pr[K_q(D_2) \in S].$$

- ϵ is the level of privacy required (privacy budget). The smaller the ϵ, the greater the privacy we have.

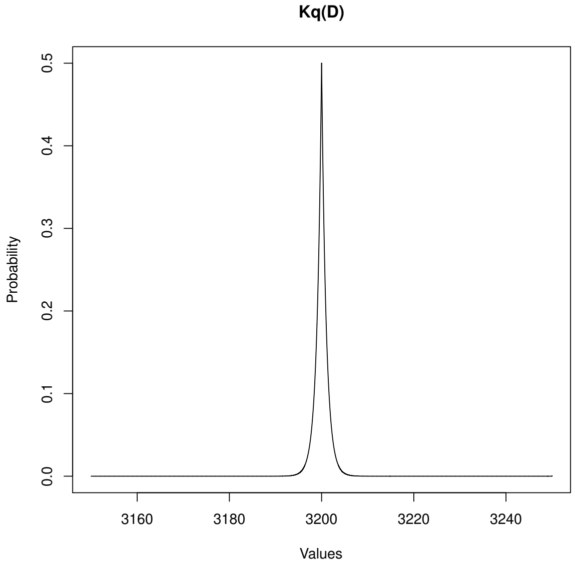
Differential privacy

- Differential privacy
  - A function K_q for a query q gives ϵ-differential privacy if . . .
    - ▷ $K_q(D)$ ~~is a constant. E.g.,~~

$$\cancel{K_q(D) = 0}$$

    - ▷ $K_q(D)$ is a randomized version of $q(D)$:

$$K_q(D) = q(D) + \textit{and some appropriate noise}$$



Differential privacy

- Properties
    - Plausible deniability: to an extend, in terms of ϵ

Differential privacy: Variations of differential privacy

- **Def. 3.17.** (ϵ, δ)-differential privacy (or δ-approximate ϵ-indistinguishability)
  - ○ A function K_q for a query q gives (ϵ, δ)-differential privacy if for all data sets D_1 and D_2 differing in at most one element, and all $S \subseteq Range(K_q)$,

$$Pr[K_q(D_1) \in S] \leq e^{\epsilon} Pr[K_q(D_2) \in S] + \delta.$$

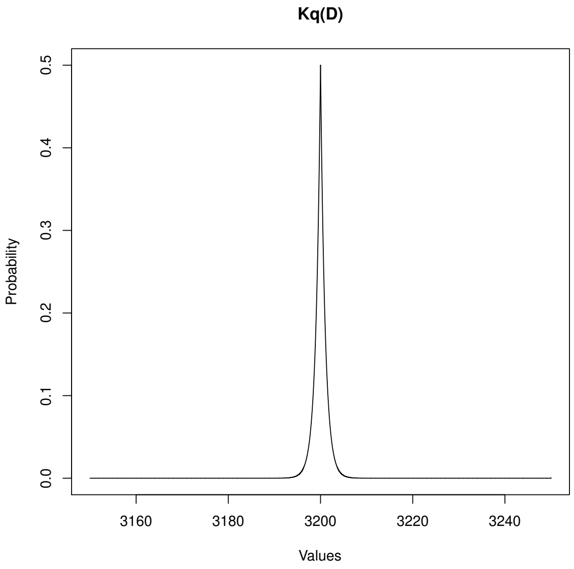
- Relaxes ϵ-DP, events with a probability smaller than δ for D_1 are still permited even if they do not occur in D_2.

Differential privacy: Variations of differential privacy

- Bounded differential privacy
  - The two neighboring datasets have exactly the same number of records.

Differential privacy: Budgets

- Multiple queries and budget consumption



Local Differential privacy

- When $q = Id$
  - That is, we want to deliver X, so, we provide $X' = \rho(X)$

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Local Differential privacy

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  - That is, we want to deliver X, so, we provide $X' = \rho(X)$
- At record level / collection level
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- Problem
  - Multiple communications from the same device:
  $$x_i^1, \ldots, x_i^T$$
  provide
  $$\rho(x_i^1), \ldots, \rho(x_i^T)$$
  but, they are not independent ...

Homomorphic encryption

Homomorphic encryption

- Homomorphic encryption. We want to avoid access to raw data and partial computations.
  - A single database DB and a function f. The only information learn is $f(DB)$. No other leakage is permitted. No access to the data, no acess to partial computations (i.e., similar to SMC but for a single database).
  - This allows us to store data in the cloud. No leakage during data storage, no leakage during transmissions.

Secure Multiparty Computation

Secure multiparty computation

- **Def. 3,18.**
  - ○ Let $P_1, \ldots, P_n$ represent a set of parties, and let $X_1, \ldots, X_n$ be their respective databases. The parties want to compute a function f of these databases (i.e., $f(X_1, \ldots, X_n)$) without revealing unnecessary information.

- After computing $f(X_1, \ldots, X_n)$ and delivering this result to each P_i, what P_i knows is nothing more than what can be deduced from X_i and the function f. So, the computation of f has not given P_i any extra knowledge.

Secure multiparty computation

- Trivial approach: Centralized approach
  - Trusted third party TTP that computes the analysis.
    Each P_i transfers data X_i using a completely secure channel (e.g., using cryptographic protocols) to the trusted third party TTP. Then, TTP computes the result $y = f(X_1, \ldots, X_n)$, and sends y to each P_i in a secure way. This will satisfy the definition as each P_i knows nothing more than X_i and y.
- Secure multiparty computation provides solutions for this problem in a distributed environment (no trusted third party). Same privacy guarantees are sought.

Secure multiparty computation

- Privacy-preserving solutions
  - Protocols that describe the information flow among the parties and details their computations.
  - Assumptions are needed on the behavior of the intruders.
    The parties themselves can be intruders trying to gain some extra knowledge from their computations. We can even consider parties that try to fool the other parties, break the protocol, and collide with others to learn relevant information from a targeted party.

Result privacy

Result privacy

- Result privacy. Given a database, avoid inferring knowledge K.
  - **Def.** X a database, A a parametric data mining algorithm. A with parameter Θ is said to have ability to derive knowledge K from X if and only if K appears either directly in the output of the algorithm or by reasoning from the output.

$$(A, X, \Theta) \vdash K$$

  - K is said to be derivable from X, if there exists any algorithm A with parameter Θ such that $(A, X, \Theta) \vdash K$.

Result privacy

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  - K is said to be derivable from X, if there exists any algorithm A with parameter Θ such that $(A, X, \Theta) \vdash K$.
  - **Def.** X, A, Θ as above. A with Θ is said to satisfy result privacy with respect to a set of sensitive knowledge $\mathcal{K} = \{K_1, \ldots, K_n\}$ when no K_i in $\mathcal{K}$ is derivable from X.
    no K_i is such that $(A, X, \Theta) \vdash K_i$.

Summary

Summary

Privacy risk	Attribute	Identity	database	query	Boolean
model/measure	disclosure	disclosure	release	release	
Re-identification		X	X		Quantitative
Uniqueness		X	X		Quantitative
Result-driven	X		X		Boolean
k-Anonymity		X	X		Boolean
k-confusion		X	X		Boolean
k-concealment		X	X		Boolean
p-sensitive k-Anonymity	X	X	X		Boolean
k-Anonymity, l-diversity	X	X	X		Boolean
k-Anonymity, t-closeness	X	X	X		Boolean
Interval disclosure	X		X		Quantitative
Differential privacy	X			X	Boolean
Local differential privacy		X	X		Boolean
Integral privacy	X			X	Boolean
Homomorphic encryption	X			X	Boolean
Secure multiparty computation	X			X	Boolean

An example: integral privacy

Integral privacy

- Given a DB and a function f, is $f(DB)$ recurrent?
  - ○ If we consider possible databases, are we going to obtain $f(DB)$ often?
  - ○ A k-anonymity flavor for $f(DB)$

Integral privacy

Some preliminaries . . .

- P the population, A be an algorithm that given a data set $S \subseteq P$ computes an output $A(S)$ that belongs to another domain $\mathcal{G}$.
- Given G in $\mathcal{G}$, previous knowledge S^* with $S^* \subset P$, the set of possible generators of G is:

$$Gen(G, S^*) = \{S' | S^* \subseteq S' \subseteq P, A(S') = G\}.$$

We use $Gen^*(G, S^*) = \{S' \setminus S^* | S^* \subseteq S' \subseteq P, A(S') = G\}$
(when no information is known on S^*, we use $S^* = \emptyset$

Integral privacy

Integral privacy, definition:

- P data, $A : S \to \mathcal{G}$, S^* background knowledge, $Gen(G, S^*)$ databases that generate G and are consistent with background knowledge S^*.
  Then, i-integral privacy is satisfied when $Gen(G, S^*)$ is large and

$$\cap_{g \in Gen^*(G,S^*)} g = \emptyset.$$

Our definition of privacy has a k-anonymity flavor (next slides)

Requirements: why? / what?

- Empty intersection to avoid all generators sharing a record (e.g., avoiding membership attacks)
- $Gen(G, S^*)$ large. What is large ????

Integral privacy

Integral privacy, details and the k-anonymity flavor

- $Gen(G, S^*)$ large . . . 1st definition
  - At least k elements, $+$ empty intersection
    $=$ k different databases not sharing records

Integral privacy

Integral privacy, details and another k-anonymity flavor

- $Gen(G, S^*)$ large . . . 2nd definition
  - ○ At least k different minimal sets

    Example. 10 databases:

    5 DBs only share record r and 5 other DBs only share record r'.

    Integrally private with $k = 2$.

  ⇒ we formalize this notion in this paper

Integral privacy

Integral privacy, and plausible deniability

- IP satisfies plausible deniability if for any record r in P such that $r \notin S^*$, there is a set/database $\sigma \in Gen^*(G, S^*)$ such that $r \notin \sigma$.

Our definition satisfies plausible deniability

Integral privacy

Integral privacy, and differential privacy

- Differential privacy, smooth function
  $A(D) \sim A(D \oplus x)$ where $D \oplus x$ means to add the record x to D
- Integral privacy, recurrent function
  If $A^{-1}(G)$ is the set of all (real) databases that can generate the output G, we require $A^{-1}(G)$ to be a large and diverse set for G.

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  If $A^{-1}(G)$ is the set of all (real) databases that can generate the output G, we require $A^{-1}(G)$ to be a large and diverse set for G.

- Simple integrally private function:
  A an algorithm that is 1 if the number of records in D is even, and 0 if the number of records in D is odd.
  That is, $f(D) = 1$ if and only if $|D|$ is even.

References

Integral privacy

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